Dr.Babasaheb Ambedkar Open University

DOR

DIPLOMA IN OPERATION RESEARCH

Block

4

Other Methods of Operation Research

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PUBLISHER :
Shri S. H. Barot
(I/C. Director & Registrar.)
Dr.Babasaheb Ambedkar Open University, Dafnala, Ahmedabad.
August, 2004 (Print)
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Printed by : Shree Neminath Printers, Shahibaug, Ahmedabad. Mobile : 9825042651

Unit : 6 : Decision Theory and Decision Trees

Introduction

The success or failure that an individual or organization experiences, depends to a large extent on the ability of making appropriate decisions. Making of a decision requires an enumeration of feasible and viable alternatives (courses of action or strategies), the projection of consequences associated with different alternatives, and a measure of effectiveness (or an objective) by which the most preferred alternative is identified. Decision theory provides an analytical and systematic approach to the study of decision-making. In other words-, decision theory provide a method of, natural decision-making wherein data concerning the occurrence of different outcomes (consequences) may be evaluated to enable the decision-maker to identify suitable alternative (or course of action).

Structure of the Chapter

6.1 Objectives:

By the end of the chapter the student will learn about

- Concept of decision theory
- Decision under uncertainty
- Decision under risk
- Decision tree analysis

6.2 Basic understanding of Decision theory:

Irrespective of the type of decision model, there are certain essential characteristics which are common to all as listed below.

Decision alternatives There is a finite number of decision alternatives available with the

decision-maker at each point in time when a decision is made. The number and type of such alternatives may depend on the previous decisions made and on what has happened subsequent to those decisions. These alternatives are also called courses of action (actions, acts or strategies) and are under control and known to the decision-maker. These may be described numerically such as, stocking 100 units of a particular item, or non-numerically such as, conducting a market survey to know the likely demand of an item.

State of nature A possible future condition (consequence or event) resulting from the choice of a decision alternative depends upon certain factors beyond the control of the decision-maker. These factors are called states of nature (future). For example, if the decision is to carry an umbrella or not, the consequence (get wet or do not) depends on what action nature takes.

The states of nature are mutually exclusive and collectively exhaustive with respect to any decision problem. The states of nature may be described numerically such as, demand of 100 units of an item or non numerically such as, employees strike, etc.

Payoff A numerical value resulting from each possible combination of alternatives and states of nature is called payoff. The payoff values are always conditional values because of unknown states of nature.

A tabular arrangement of these conditional outcome (payoff) values is known as payoff matrix as shown in following Table

General Form of Payoff Matrix

	Courses of Action (Alternatives)		
States of Nature	S_1	S_2	S_n
N_1	P_{11}	P_{12}	P_{1n}
N_2	P_{21}	P_{22}	P_{2n}
N_m	P_{m1}	P_{m2}	p_{mn}

6.3 Steps In Decision Theory Approach

The decision-making Activity involves the following steps:

- Identify and define the problem
- Listing of all possible future events, called states of nature, which can occur in the context of the decision problem.
- Such events are not under the control of decision-maker because these are erratic in nature
- Identification of all the courses of action (alternatives or decision choices) which are available to the decision-maker.
- The decision-maker has control over these courses of action,

- Expressing the payoffs (p_{ij}) resulting from each pair of course of action and state of nature. These payoffs are normally expressed in a monetary value,
- Apply an appropriate mathematical decision theory model to select best course of action from the given list on the basis of some criterion (measure of effectiveness) that results in the optimal (desired) payoff.

6.4 Types Of Decision Making Environments

Decisions are made based upon the data available about the occurrence of events as well as the decision situation (or environment). There are four types of decision-making environment: Certainty, uncertainty, risk and. conflict.

Type 1 Decision Making under Certainty

In this case the decision-maker has the complete knowledge (perfect information) of consequence of every decision choice (course of action or alternative) with certainty. Obviously, he will select an alternative that yields the largest return (payoff) for the known future (state of nature).

Type 2 Decision Making under Risk

In this case the decision-maker has less than completes knowledge with certainty of the consequence of every decision choice (course of action). This means there is more than one state of nature (future) and for which he makes an assumption of the probability with which each state of nature will occur.

Type 3 Decision Making under Uncertainty

In this case the decision-maker is unable to specify the probabilities with which the various states of nature (futures) will occur. Thus, decisions under uncertainty are taken with even less information than decisions under risk.

6.5 Decision Making Under Uncertainty

In the absence of knowledge about the probability of any state of nature (future) occurring, the decision-maker must arrive at a decision only on the actual conditional payoff values, together with a policy (attitude). There are several different criteria of decision-making in this situation. The criteria that we will discuss in this section include

(i) Maximax or Minimin

(ii) Maximin or Minimax

(iii) Equally likely

(iv) Criterion of realism

(v) Criterion of regret

(i) Maximin and Maximax Criteria

The maximin criterion is based upon the 'conservative approach' to assume that the worst possible is going to happen. The decision-maker considers each alternative and locates the minimum payoff for each ; and then selects that alternative which maximizes the minimum payoff :

Step 1. Determine the minimum assured payoff for each alternative.

Step 2. Choose that alternative which corresponds to the maximum of the above minimum payoffs.

The maximam criterion is based upon 'extreme optimism'. The decision-maker selects that particular strategy which corresponds to the maximum of the maximum payoff for each strategy:

Step 1. Determine the maximum possible payoff for each alternative.

Step 2. Select that alternative which corresponds to the maximum of the above maximum payoffs.

(ii) The Hurwicz Criterion

In order to overcome the disadvantages of extreme pessimism of 'maximin' and extreme optimism of 'maximax' criterion, L. Hurwicz introduced the concept of coefficient of optimism (or pessimism). This concept allows the decision-maker to take into account both the maximum and minimum payoffs for each alternative and assign them weights according to his degree of optimism (or pessimism). The alternative which maximises the sum of these weighted payoffs is then selected :

Step 1. Choose an appropriate degree of optimism (or pessimism) of the decision-maker. Let a be his degree of optimism (so that 1 - α is his degree of pessimism).

Step 2. Determine the maximum as well as minimum payoffs for each alternative and obtain the quantities

h = α x maximum + (1 - α) x minimum

for each alternative.

(iii) The Laplace Criterion

The Laplace criterion uses all the information by assigning equal probabilities to the possible payoffs for each action and then selecting that alternative which corresponds to the maximum expected payoff:

Step 1. Assign equal probabilities (1/n) to each payoff of a strategy (having n possible payoffs).

Step 2. Determine the expected payoff value for each alternative.

Step 3. Select that alternative which corresponds to the maximum of the above expected payoffs.

(iv) The Minimam Regret Criterion

The minimax regret criterion is based upon the assumption that the decision-maker might experience 'regret' after he has made the decision and the events have occurred. The decision-maker would like to select that alternative which will minimise his maximum possible regret:

Step 1. Determine the amounts of regret for payoff of each alternative for a particular event. The regret amount for the fth alternative when event J occurs is given by

i th regret = (maximum payoff- i th payoff) for the jth event.

Step 2. Determine the maximum regret amount for each alternative.

Step 3. Choose that alternative which corresponds to the minimum of the above

maximum regrets.

The Emv and Eol Criterion

In most of the business situations, there is risk and uncertainty. The decision-maker cannot predict as to what outcome (payoff) will occur as a consequence of his choosing a particular act. At the most he can write (subjectively, or on the basis of past experience) the various outcomes (payoffs) that are likely to occur, along with their corresponding probabilities of occurrence. The expected payoff (profit), associated with a given combination of act and event, is obtained by multiplying the payoff for that act-event combination by the probability of occurrence of the given event. The expected monetary value (EMV) of an act is the sum of all expected conditional profits associated with that act. The EMV Criterion can be summarized as below :

Step 1. List the conditional profit for each act-event combination, along with the corresponding event probabilities.

Step 2. For each act, determine the expected conditional profits.

Step 3. Determine EMV for each act.

Step 4. Choose the act which corresponds to the optimal EMV.

The decision-maker might experience regret after the decision has been taken and the events have occurred. For a given event, the difference between the payoff value of the most favourable act (having highest payoff) and some other act may be termed 'loss' due to losing the 'opportunity' of choosing the most favourable act. We call this loss as the conditional opportunity loss (COL) for that act. The EOL Criterion may be summarised as below :

Step 1. List the conditional profit table for each act-event combination, along with corresponding event probabilities.

Step 2. For each event, determine the COL values by first locating the most favourable act (maximum payoff) for that event and then taking the difference between that conditional profit value and each conditional profit value for that event.

Step 3. For each act, determine the expected COL values and sum these values to get the expected opportunity loss (EOL) for that act.

Step 4. Choose that act which corresponds to the minimum COL value.

6.6 Expected Value of Perfect Information (EVPI)

In decision-making under risk each state of nature is associated with the probability of its occurrence. However, if the decision-maker can acquire perfect (complete and accurate) information about the occurrence of various states of nature, then he will be able to select a course of action that yields the desired payoff for whatever state of nature that actually occurs.

We have seen that the EMV or EOL criterion helps the decision-maker select a particular course of action .that optimizes the expected payoff without any additional information. Expected value of perfect "Information (EVPI) represents the maximum amount of money the decision-maker has to pay to get this additional information about the occurrence of various states of nature before a decision has to be made. Mathematically it is stated as

EVPI = Expected profit (or value) with perfect information under certainty - Expected profit without perfect information

$$= \sum_{I=1}^{m} p_{ij} (N_i)p_i - EMV^*$$

where p_{ij} = best payoff for state of nature, N_i

p_i = probability of state of nature, N_i

EMV* = maximum expected monetary value

6.7 Decision Tree Analysis

A decision tree is a graphic display of various decision alternatives and the sequence of events as if they were branches of a tree. In constructing a tree diagram, it is a convention to use the symbol O' to indicate the decision point and 'O' to denote the situation of uncertainty or 'event'. Branches coming out of a decision point are nothing but representation of immediate mutually exclusive alternative acts (alternative options) open to the decision-maker. Branches emanating from the 'event' point 'O' represent all possible situations (events). These events are not fully under the control of the decision-maker and may represent consumer demand, etc. The basic advantage of a tree diagram is that another act (called second act) subsequent to the happening of each event may also be represented. The resulting outcome (Payoff) for each act-event combination may be indicated in the tree diagram at the outer end of each branch. A decision-tree diagram is illustrated below :

When probabilities of various events are known, they are written along the corresponding branches. Multiplying the probabilities along the branches results in the joint probabilities for the corresponding act-event sequence. Thus in a decision-tree, the decision maker lists all possible alternatives, possible events and resulting payoff values along with their probabilities for each act-event sequence. This enables him to determine expected payoff values and hence the EMV for each act.

A decision-tree is highly useful to a decision-maker in multistage situations which involve a series of decisions each dependent on the preceding one. The modern approach to the analysis of a decision tree involves the computation of EMV for each main branch of the tree. These values then become the conditional expected payoffs for their respective branches; the process is repeated until the EMV for a given path has been determined. The optimal path (strategy) is one that corresponds to the maximum EMV. This technique, called Rolling Back Technique, is explained below :

Since It is impossible to evaluate an Immediate decision act without first considering all future outcomes that result from this decision, one begins the analysis at the end of the tree (far right hand side). The last decision point is of primary importance to us. We analyse this point and take that decision which yields optimal EMV and then roll back to the last but one decision point, make the same EMV analysis for decision and roll-back to the preceding decision point. The rolling back process continues till the initial decision point is reached.

6.8. Exercise:

1. Describe the concept of decision making?
2. State the types of environment under which decisions can be made?
3. Write a short note on Expected Value of Perfect Information?
4. Write a short note on Expected Opportunity Loss?
5. Write a short note on maximax criteria of decision making?
6. Explain the decision tree approach in decision making?

1. Abdul, a food products company is contemplating the introduction of a revolutionary new product with new packaging or replace the existing product at much higher price (S1) or a moderate change in the -composition of the existing product with a new packaging at a small increase in price (S_2) or a small change in the composition of the existing product except the word 'New' with a negligible increase in price (S_3). The three possible states of nature or events are: (i) high increase in sales (N1), (ii) no change in sales (N2) and (iii) decrease in sales (N_3). The marketing department of the company worked out the payoffs in terms of yearly net profits for each of the strategies of three events (expected sales). This is represented in the following table:

	States of Nature		
Strategies	N1	N_2	N3
S1	7,00,000	3,00,000	1,50,000
S_2	5,00,000	4,50,000	0
S_3	3,00,000	3,00,000	3,00,000

Which strategy should the concerned executive choose on the basis of

(i) Maximin criterion (ii) Maximax criterion

(iii) Minimax regret criterion (iv) Laplace criterion?

Solution The payoff matrix is rewritten as follows:

(i) Maximin Criterion

States of		Strategies		
Nature	S1	S_2	S_3	
N1	7,00,000	5,00,000	3,00,000	
N_2	3,00,000	4,50,000	3,00,000	
N3	1,50,000	0	3,00,000	
Column minimum 1,50,000		0	3,00,000	Maximin

The maximum of column minima is 3,00,000. Hence, the company should adopt strategy S_3

(ii) Maximax Criterion

	Strategies		
States of Nature	S1	S_2	S3
N1	7,00,000	5,00,000	3,00,000
N_2	3,00,000	4,50,000	3.00,000
N3	1,50,000	0	3,00,000
Column maximum 7,00,000		5,00,000	3,00,000
Maximax			

The maximum of cplumn maxima is 7,00,000. Hence, the company should adopt strategy Sj.

(iii) Miriimax Regret Criterion Opportunity loss table is shown below:

States of Nature	Strategies		
	S1	S_2	S3
N!	7,00,000 - 7,00,000	7,00,000 - 5,00,000	7,00,000 - 3,00,000
	= 0	= 2,00,000	= 4,00,000
N2	4,50,000 - 3,00,000	4,50,000 - 4,50,000	4,50,000 - 3,00,000
	= 1,50,000	= 0	= 1,50,000
N3	3,00,000- 1,50,000	3,00,000 - 0	3,00,000 - 3,00,000
	= 1,50,000	= 3,00,000	= 0
Column maximum 1,50,000		3,00,000	4,00,000
Minimax regret			

Hence the company should adopt minimum opportunity loss strategy, 5j.

(iv) Laplace Criterion Since we do not know the probabilities of states of nature, assume that they are equal. For this example, we would assume that each state of nature has a probability 1/3 of occurrence. Thus,

Strategy	Expected Return (Rs)
S1	.(7,00,000 + 3,00,000 + 1,50,000)73 = 3,83,333.33
S_2	(5,00,000 + 4,50,000 + 0)/3 = 3,16,666.66
S_3	(3,00,000 + 3,00,000 + 3,00,000)/3 = 3,00,000

Since the largest expected return is from strategy S_1, the executive must select strategy S_1.

2. Jaydip, a manufacturer makes a product, of which the principal ingredient is a chemical X. At the moment, the manufacturer spends Rs 1,000 per year on supply of X, but there is a possibility that the price may soon increase to four times its present figure because of a worldwide shortage of the chemical. There is another chemical Y, which the manufacturer could use in conjunction with a third chemical, Z in order to give the same effect as chemical X. Chemicals Y and Z would together expense the manufacturer Rs 3,000 per year, but their prices are unlikely to rise. What action should the manufacturer take? Apply the maximin and minimax criteria for decision-making and give two sets of solutions. If the coefficent of optimism is 0.4, find the course of action that minimizes the expense.

Solution The data of the problem is summarized in the following table (negative figures in the table represents profit).

States of Nature	Courses of Action	
	S1 (use Y and Z)	S_2 (use X)
N_1 (Price of X increases)	- 3,000	- 4,000
N_2 (Price of X does not increase)	-3,000	-1,000

(i) Maximin Criterion

States of Nature	Courses of Action	
	S1	S_2
N1	-3,000	- 4,000
N2	- 3,000	-1,000
Column minimum - 3,000 Maximin		-4,000

Maximum of column minima = - 3,000. Hence, the manufacturer should adopt action S1.

(ii) Minimax (or opportunity loss) Criterion

States of Nature		Courses of Action		
		S1	S_2	
	N1	- 3,000 - (- 3,000) = 0	- 3,000 - (- 4,000) =	1,000
	N_2	-1,000- (-3,000) = 2,000	-1,000 -(-1,000) =	0
Maximum	opportunity	2,000	1,000 <- Minimax	

Hence, manufacturer should adopt minimum opportunity loss course of action S_2.

(iii) Hurwicz Criterion Given the coefficient of optimism equal to 0.4, the coefficient of pessimism will be 1 - 0.4 = 0.6. Then according to Hurwicz, select course of action that

optimizes (maximum for profit and minimum for expense) the payoff value

$H = \alpha$ (Best payoff) + (1 - α) (Worst payoff) = α (Maximum in column) + (1 - α) (Minimum in column)			
Course of Action	Best Payoff,	Worst Payoff	H
S1	-3,000	-3,000	-3,000
S_2	- 1,000	-4,000	- 2,800

Since course of action S2 has the least expense (maximum profit) = 0.4(1,000) + 0.6(4,000) = Rs 2,800, the manufacturer should adopt it.

3. Mr X quite often flies from town A to town B. He can use the airport bus which expenses Rs. 25 but if he takes it, there is a 0.08 chance that he will miss the flight. The stay in a hotel expenses Rs 270 with a 0.96 chance of being on time for the flight. For Rs 350 he can use a taxi which will make 99 per cent chance of being on time for the flight. If Mr X catches the plane on time, he will conclude a business transaction which will produce a profit of Rs 10,000, otherwise he will lose it. Which mode of transport should Mr X use? Answer on the basis of the EMV criterion.

Solution Computation of EMV of various courses of action is shown in following table.

States of Nature	Bus			Courses of Action Stay in Hotel			Taxi		
	Expense	Prob.	Expected Value	Expense	Prob.	Expected Value	Expense	Prob.	Expected Value
Catches the flight	10,000 25= 9,975	0.92	9,177	10,000 - 270 = 9,730	0.96	9.340.80	10,000 - 350 = 9,650	0.99	9,553.50
Miss the flight	-25	0.08	-2.0	-270	0.04	-10.80	-350	0.01	-3.50
Expected monetary value (EMV)			9,175			9,330			9,550

Comparing the EMV associated with each course of action indicates that course of action 'Taxi' is the logical alternative because it has the highest EMV.

4. The probability of the demand for lorries for hiring on any day in a given district is as follows:

No. of lorries demanded :	0	1	2	3	4
Probability :	0.1	0.2	0.3	0.2	0.2

Lorries have a fixed expense of Rs 90 each day to keep the daily hire charges (net of variable expenses of running) Rs 200. If the lorry-hire company owns 4 lorries, what is its daily expectation? If the company is about to go into business and currently has no lorries, how many lorries should it buy?

Solution It is given that Rs 90 is the fixed expense and Rs 200 is variable expense. Now the payoff values with 4 lorries at the disposal of decision-maker are calculated as under:

No. of Lqrries demanded :	0	I	2	3	4
Payoff :: (with 4 lorries)	0 - 90 × 4 = -360	200 - 90 × 4 = -160	400 - 90 × 4 = 40	600 - 90 × 4 = 240	800 - 90 × 4 = 440

Thus daily expectation is obtained by multiplying the payoff values with the given **corresponding probabilities** of demand:

Daily Expectation = (- 360)(0.1) + (- 160)(0.2) + (40)(0.3) + (240)(0.2) + (440)(0.2) = Rs 80 The conditional payoffs and expected payoffs for each course of action are shown in following tables.

Conditional Payoff Values

Demand of Lorries	Probability	Conditional Payoff (Rs) due to Decision to Purchase Lorries (Course of Action)				
		0	1	2	3	4
0	0.1	0	-90	-180	-270	-360
1	0.2	0	110	20	-70	-160
2	0.3	0	110	220	130	40
3	0.2	0	110	220	330	240
4	0.2	0	110	220	330	440

Expected Payoffs and EMV

Demand of Lorries	Probability	Conditional Payoff (Rs) due to Decision to Purchase Lorries (Course of Action)				
		0	1	2	3	4
0	0.1	0	-9	-18	-27	-36
1	0.2	0'	22	4	-14	-32
2	0.3	0	33	66	39	12
3	0.2	0	22	44	66	48
4	0.2	0	22	44	66	88
EMV		0	90	140	130	80

Since EMV Rs 140 for the course of action 2 is highest, the company should buy 2 lorries.

5. Alkesh Ltd. manufactures goods for a market in which the technology of the product is 'changing rapidly. The research and development department has produced a new product which appears "to have potential for commercial exploitation. A further Rs 60,000 is required for development testing.

The company has 100 customers and each customer might purchase at the most one unit of the product. Market research suggests that a selling price of Rs 6,000 for each unit with total variable expenses of manufacturing and selling estimate are Rs 2,000 for each unit.

From previous experience, it has been possible to derive a probability distribution relating to the proportion of customers who will buy the product as follows:

Proportion of customers :	0.04	0.08	0.12	0.16	020
Probability :	0.10	0.10	0.20	0.40	0.20

Determine the expected opportunity losses, given no other information than that stated above, and state whether or not the company should develop the product.

Solution If p is the proportion of customers who purchase the new product, the conditional profit is: (6,000 - 2,000) × 100 p - 60,000 = Rs (4,00,000 p - 60,000)

Let Ni (i =.1,2, ..., 5) be the possible states of nature, i.e. proportion of the customers who will buy the new product and S1 (develop the product) and S_2 (do not develop the product) be the two courses of action.

The conditional profit values (payoffs) for each pair of Ni's and Sj's are shown in following table.

Conditional Profit Values (Payoffs)

State of Nature (Proportion of Customers, p)	Conditional Profit = Rs (4,00,000 p - 60,000) Course of Action	
	S1 (Develop)	S2 (Do not Develop)
0.04	- 44,000	0
0.08	-28,000	0
0.12	- 12,000	0
0.16	4,000	0
0.20	20,000	0

Opportunity loss values are shown in following table.

Opportunity Loss Values

State of Nature	Probability	Conditional Profit (Rs)		Opportunity Loss (Rs)	
		S1	S_2	S1	S_2
0.04	0.1	- 44,000	0	44,000	0
0.08	0.1	- 28,000	0	28,000	0
0.12	0.2	- 12,000	0	12,000	0
0,16	0.4	4,000	0	0	4,000
0.20	0.2	20,000	0	0	20,000

Using the given estimates of probabilities associated with each state of nature, the expected opportunity loss (EOL) for each course of action is given below:

EOL (S1) = 0.1 (44,000) + 0.1 (28,000) + 0.2 (12,000) + 0.4 (0) + 0.2 (0) = Rs 9,600
EOL (S_2) = 0.1 (0) + 0.1 (0) + 0.2 (0) + 0.4 (4,000) + 0.2 (20,000) = Rs 5,600

Since the company seeks to minimize the expected opportunity loss, the company should select course of action S_2 (do not develop the product) with minimum EOL.

6. Padmavati Ltd. needs to increase its production beyond its existing capacity. It has narrowed the alternatives to two approaches to increase the production capacity: (a) expansion, at a expense of Rs 8 -million, or (b) modernization at a expense of Rs 5 million. Both approaches would require the same amount of time for implementation. Management believes that over the required payback period, demand will either be high or moderate. Since high demand is considered to be somewhat less likely than moderate demand, the probability of high demand has been set at 0.35. If the demand is high, expansion would gross an estimated additional Rs 12 million but modernization only an additional Rs 6 million, due to lower maximum production capability. On the other hand, if the demand is moderate, the comparable figures would be Rs 7 million for expansion and Rs 5 million for modernization.

.(a) Calculate the conditional profit in relation to various action-and-outcome combinations and states of nature-.

(b) If the company wishes to maximize its expected monetary value (EMV), should it modernize or expand?

(c) Calculate the EVPI.

(d) Construct the conditional opportunity loss table and also calculate EOL.

Solution (a) Defining the states of nature: High demand and Moderate demand (over which the company has no control) and courses of action (company's possible decisions): Expand and Modernize.

Since the probability that the demand is high estimated at 0.35, the probability of moderate demand must be (1 - 0.35) = 0.65. The calculations for conditional profit values are shown in Table 11.10.

(c) To calculate EVPI, we shall first calculate EPPI. For calculating EPPI, we choose optimal course of action for each state of nature, multiply its conditional profit by the given probability to get weighted profit, and then sum these weights as shown in following table.

State of Nature (Demand)	Probability	Optimal Course of Action	Profit from Optimal Course of Action (Rs million)	
			Conditional Profit	Weighted Profit
High demand	0.35	$^{5}1$	4	4 × 0.35 = 1.40
Moderate demand	0.65	S_2	0	0 × 0.65 = 0
				EPPI = 1.40

The optimal EMV* is Rs 0.75 million corresponding to the course of action S1. Then

EVPI =EPPI-EMV(S1)

= 1.40-0.75 = Rs 0.65 million.

In other words, if the company could get a perfect information (or forecast) of demand (high or moderate), it should consider paying up to Rs 0.65 million for an information.

The expected value of perfect information in business helps in getting an absolute upper bound on the amount that should be spent to get additional information on which to base a given decision.

(d) The opportunity loss values are shown in following table.

State of Nature (Demand)	Probability	Conditional Profit (Rs million) due to Course of Action		Conditional Opportunity Loss (Rs million) due to Course of Action	
		S1	S_2	S1	S_2
High demand (N1)	0.35	4	1	0	3
Moderate demand (N_2)	0.65	-1	0	1	0

The conditional opportunity loss values may be explained as follows: If high demand (TV,) occurred, then the maximum profit, of Rs 4 million would be achieved by selecting course of action S_t. Therefore, the selection of S_1 would result in zero opportunity loss, as it is the best decision that can be made if TV, occurs. If course of action S_2 was chosen with a payoff of one million, then this would result in an opportunity loss of 4 - 1 = Rs 3 million. If moderate demand (W_2) occurred, then the best course of action would be S_2 with Rs zero million profit. Thus, opportunity loss would be associated with the selection of S_2 but if S1 was selected, then the opportunity loss would be 0 — (— 1) = Rs 1 million. That is, the company would have been Rs 1 million worse off if it had chosen course of action S_2.

Using the given estimates of probabilities associated with each state of nature, i.e. P(N1) = 0.35, and $P(N_2)$ = 0.65, the expected opportunity losses for the two courses of action are:

EOL(S1) = 0.35(0) + 0.65(1) = Rs 0.65 million

$EOL(S_2)$ = 0.35(3) + 0.65(0) = Rs 1.05 million

Since decision-maker seeks to minimize the expected opportunity loss, he must select course of action S, as it produces the smallest expected opportunity loss.

7. PQR Ltd. manufactures parts for passenger cars and sells them in lots of 10,000 parts each. The company has a policy of inspecting each lot before it is actually shipped to the retailer. Five Inspection categories, established for quality control, represent the percentage of defective items contained in each lot. These are given in the following table. The daily inspection chart for past 100 inspections shows the following rating or breakdown inspection:

The management is considering two possible courses of action:

(i) S1 Shut down the entire plant operations and thoroughly inspect each machine.

Rating	Proportion of Defective Items	Frequency
Excellent (A)	0.02	25
Good (B)	0.05	30
Acceptable (C)	0.10	20
Fair (D)	0.15	20
Poor (E)	0.20	5
		Total = 100

(ii) S_2: Continue production as it now exists but offer the customer a refund for defective items that are discovered and subsequently returned.

The first alternative will expense Rs 600 while the second alternative will expense the company Re 1 for each defective item that is returned. What is the optimum decision for the company? Find the EVPI.

Solution Calculations of inspection and refund expense are shown in following table.

Inspection and Refund Expense

Rating	Defective Rate	Probability	Expense		Opportunity Loss	
			Inspect	Refund	Inspect	Refund
A.	0.02	0.25	600	200	400	0
B.	0.05	0.30	600	500	100	0
C	0.10	0.20	600	1,000	0	400
D	0715	0.20	600	1,500	0	900
E	0.20	0.05	600	2,000	0	1,400
		1.00	600*	670	170*	240

The expense of refund is calculated as follows: For lot A : 10,000 × 0.02 × 1.00 = Rs 200

Similarly, the expense of refund for other lots is calculated. Expected expense of refund is:

200 × 0.25 + 500 × 0.30+ ...+2,000-× 0.05 = Rs 670

Now expected expense of inspection is:

600 × 0.25 + 600 × 0.30 + ... + 600 × 0.05 = Rs 600

Since the expense of refund is more than the expense of inspection, the plant should be shut down for inspection. 'Also, EVPI = EOL of inspection = Rs 170.

8. Ram, a farmer is attempting to decide which of three crops he should plant on his 100 acre farm. The profit from each crop is strongly dependent on the rainfall during the growing season. He has categorized the amount of the rainfall as substantial, moderate or light. He estimates his profit for each crop as shown in the table below:

Rainfall	Estimated Profit (Rs)		
	Crop A	Crop B	Crop C
Substantial	7,000	2,500	4,000
Moderate	3,500	3,500	4,000
Light	1,000	4,000	3,000

Based on the weather in previous seasons and the current projection for the coming season, he estimates the probability of substantial rainfall as 0.2, that of moderate rainfall as 0.3 and that of light rainfall as 0.5.

Furthermore, services of forecasters could be employed to provide a detailed survey of current rainfall prospects as shown in the table.

Rainfall	Estimated Profit (Rs)		
	Crop A	Crop B	Crop C
Substantial	0.70	0.25	0.05
Moderate	0.30	0.60	0.10
Light	0.10	0.20	0.70

(a) From the available data, determine the optimal decision as to which crop to plant. ...

(b) Determine whether it would be economical for the farmer to hire the services of a forecaster.

Solution (a) Let N_i be the state of nature (i = 1,2, 3) representing 'substantial rainfall', 'moderate rainfall' and 'light rainfall', respectively and Sj be the course of action (j = 1, 2, 3) representing 'Crop A', 'Crop B' and 'Crop C', respectively.

Calculation of EMVs

States of Nature	Prior Probability	Conditional Profit (Rs) Courses of Action			Expected Profit (Rs) Courses of Action		
		S1	S_2	S3	S1	S_2	S_3
N1	0.2	7,000	2,500	4,000	1,400	500	800
N2	0.3	3,500	3,500	4,000	1,050	1,050	1,200
N3	0.5	1,000	4,000	3,000	500	2,000	1,500
					EMV = 2,950	3,550	3,500

The maximum EMV is Rs 3,550. Therefore, optimal course of action is $N_{2>}$ i.e. plant crop B. However, it would make no sense to plant more than one kind of crop because maximum EMV is obtained by planting all 100 acres with crop B.

(b) Let Bi (i = 1,2,3) denote the outcome forecast for 'substantial rainfall', 'moderate rainfall' and 'light rainfall' respectively. The likelihood values are given in following table.

where maximum profit for each state of nature is written in bold, the expected profit with perfect information is given by

$$EPPI = 0.2(7,000) + 0.3(4,000) + 0.5(4,000) = Rs\ 4,600$$

Thus, we have EVPI = EPPI - EMV* = 4,600 - 3,550 = Rs 1,050

For each of the three forecast results, the prior and posterior probabilities are given in following tables.

States of Nature	Prior Probability	Outcomes (Bi)	Conditional Probability $P(B_i \mid Ni)$	Joint Probability P(Bi Ni) = P(Ni) P(Bi | Ni)		
N1	0.2 .	B1	0.70	0.14	—	—
		B_2	0.25	—	0.05	—
		B3I	0.05	—	—	0.01
N2	0.3	B1	0.30	0.09	—	—
		B_2	0.60	—	0.18	—
		B3I	0.10		—	0.03
N#	0.5	B1	0.10	0.05	—	—
		B_2	0.20	—	0.10	—
		B3I	0.70	—	—	0.35
Marginal probability				0.28	0.33	0.39

Outcome	Probability	States of Nature Posterior Probability		
(Bi)	P(Bi)		P(Ni | Bi) = P(Ni | Bi.) |P(Bi)	
B1	0.28	N1	0.14/0.28 =	0.500
		N2	0.09/0.28 =	0.321
		N3	0.05/0.28 =	0.178
B2	0.33	N1	0.05/0.33 =	0.151
		N2	0.18/0.33 =	0.303
		N3	0.10/0.33 =	0.030
B#	0.39	N1	0.01/0.39 =	0.025
		N2	0.03/0.39 =	0.076
		N3	0.35/0.39 =	0.897

For each outcome, the revised probabilities are now used to recalculate the EMVs given the additional information supplied by that outcome, as shown in following table.

States of Nature (Ni)	BI			Forecast Outcome B_2			BI3		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N1	0.500	0	0	0.151	500	75	0.025	3,000	60
N2	0.321	4,500	1,440	0.303	500	275	0.076	0	0
N3	0.178	3.000	540	0.030	0	0	0.897	1,000	900
Posterior	EOL		1,980			350			960

The expected value of sample information can be obtained by multiplying posterior EOLs with revised probabilities as shown in following table.

Outcome Bi	Probability P(Bi)	Expected Opportunity Loss (EOL)	Expected Value of Sample Information (EVSI)
BI	0.28	1980	554.4
B_2	0.33	350	115.5
B3	0.39	960	374.4
			Total 1044.3

The EVSI Rs 1,044.3 indicates the money which the farmer has to pay for hiring the services of a forecaster.

9. Anubhav, a glass factory specializing in crystal is developing a substantial backlog and the firm's management is considering three courses of action: Arrange for sub-contracting (S1), begin overtime production (S_2) and construct new facilities (S_3). The correct choice depends largely upon future demand which may be low, medium, or high. By consensus, management ranks the respective probabilities as 0.10, 0.50 and 0.40. A. expense analysis reveals effect upon the profits that is shown in the table below:

Demand	Probability		Course of Action	
		S1 (Subcontracting)	S2 (Begin Overtime)	S_3 (Construct Facilities)
Low (L)	0.10	10	-20	-150
Medium (M)	0.50	50	60	20
High(H)	0.40	50	100	200

Show this decision situation in the form of a decision tree and indicate the most preferred decision and corresponding expected value.

Solution A decision tree which represents possible courses of action and states of nature are shown in Given Figure. In order to analyze the tree, we start working backward from the end branches.

The most preferred decision at the decision node 0 is found by calculating expected value of each decision branch and selecting the path (course of action) with high value.

Since node 3 has the highest EMV, therefore, the decision at node 0 will be to choose the course of action S_3, i.e. construct new facilities.

10. The Oil India Corporation (OIC) is considering whether to go for an offshore oil drilling Contract to be awarded in Bombay High. If OIC bid, value would be Rs 600 million with a 65% chance of gaining the contract. The OIC may set up a new drilling operation or move already existing operation, which has proved successful, to a new site. The probability of success and expected returns are as follows:

Outcome	New Drilling Operation		Existing operation	
	Probability	Expected Revenue (Rs million)	Probability	Expected Revenue (Rs million)
Success	0.75	800	0.85	700
Failure	0.25	200	0.15	350

If the Corporation do not bid or lose the contract, they can use Rs 600 million to modernize their operation. This would result in a return of either 5 per cent or 8 per cent on the sum invested with probabilities 0.45 and 0.55. (Assume that all expenses and revenue have been discounted to present value.)

(a) Construct a decision tree for the problem showing clearly the courses of action.

(a) By applying an appropriate decision criterion recommend whether or not the Oil India Corporation should bid the contract.

Solution The decision tree based on the given information in the problem is depicted in the given figure in table given below

Evaluation of Decision and Chance Nodes

Decision Point	Outcome	Probability	Conditional Value	Expected Value (Rs)
D_3 (i) Modernize	5% return	0.45	600 × 0.05 = 30	13.5
	8% return	0.55	600 × 0.08 = 48	26.4
				Rs 39.9
D_2 (i) Undertake new	Success	0.75	800	600
drilling operation	Failure	0.25	200	50
				Rs 650
(ii) Move existing	Success	0.85	700	595

operation	Failure	. 0.15	350	52.5
				Rs 647.5
Di (i) Do not bid but	5% return	0.45	600 × 0.05 = 30	13.5
modernize	8% return	0.55	600 x 0.08 = 48	26.4
				Rs 39.9
(ii) Bid .	Success	0.65	650	422.50
	Failure	0.35	39.9	13.96
				Rs 436.46

Since EMV, Rs 257.34 at node 2 is highest, therefore the best decision at node D_2 is to decide for bid and if successful establish a new drilling operation.

Practical Exercise:

1. You are given the following payoffs of three acts A1, A1, and A_3 and the events E1, E_2, E_3.

States of .Nature	Three Acts		
	A1	A2	A3
E_1	25	-10	-125
E_2	400	440	400
E3	650	740	750

The probabilities of the states of nature are respectively 0.1, 0.7 and 0.2. Calculate and tabulate EMV and conclude 'which of the course of action can be chosen as the best.

2. In Dar Ltd., management is faced with the problem of choosing one of three products for manufacturing. The potential demand for each product may turn out to be good, moderate or poor. The probabilities for each of the states of nature were estimated as follows:

Product	Nature of Demand		
	Good	Moderate	Poor
X	0.70	0.20	0.10
Y	0.50	0.30	0.20
Z	0.40	0.50	0.10

The estimated profit or loss in rupees under the three states may be taken as:

Product	Good	Moderate	Poor
X	30,000	20,000	10,000
Y	60,000	30,000	20,000
Z	40,000	10,000	- 15,000

Prepare the expected value table, and advise the management about the choice of product.

3. The marketing staff of Kash Ltd. has submitted the following payoff table, giving profits in million rupees, concerning a certain proposal depending upon the rate of technology advance.

Technological Advance		Decision
	Accept	Reject
Much	2	3
Little	5	2
None	(-)I	4

The probabilities are 0.2, 0.5 and 0.3 for Much, Little and None technological advance respectively. What decision should be taken?

4. Akash, a physician purchases a particular vaccine on Monday each week. The vaccine must be used within the following week, otherwise it becomes worthless. The vaccine expenses Rs 2 per dose and the physician charges Rs 4 per dose. In the past 50 weeks, the physician has administered the vaccine in the following quantities:

Doses per week :	20	25	40	60
Number of weeks :	5	15	25	5

Determine how many doses the physician should buy every week.

5. A grocery with a Abdul bakery department is faced with the problem of how many cakes to buy in order to meet the day's demand. The grocer prefers not to sell day-old goods in competition with fresh products; leftover cakes are, therefore, a complete loss. On the other hand, if a customer desires a cake and all of them have been sold, the disappointed customer will buy from elsewhere and the sales will be lost. The grocer has, therefore, collected information on the past sales on a selected 100-day period as shown in table below:

Sales per Day	No. of Days	Probability
25	10	0.10
26	30	0.30
27	50	0.50
28	10	0.10

Construct the payoff table and the opportunity loss table. What is the optimal number of cakes that should be bought each day? Also find and interpret EVPI (Expected Value of

Perfect Information). A cake expenses Re 0.80 and sells for Re 1.

6. Anup, a producer of boats has estimated the following distribution of demand for a particular kind of boat:

No. demanded :	0	1	2	3	4	5	6
Probability .:	0.14	0.27	0.27	0.18	0.09	0.04	0.01

Each boat expenses him Rs 7,000 and he sells them for Rs 10,000 each. Boats left unsold at the end of the season must be disposed of for Rs 6,000 each. How many boats should be in stock so as to maximize his expected

profit?

7. Akash, a small industry finds from the past data that the expense of making an item is Rs 25, the selling price is Rs 30 if it is sold within a week, and it could be disposed of at Rs 20 per item at the end of the week:

Weekly sales	: < 3	4	5	6	7	> 8
No. of weeks	: 0	10	20	40	30	0

Find the optimum number of items per week the industry should produce.

8. AB & co., a firm makes pastries which it sells at Rs 8 per dozen in special boxes containing one dozen each. The direct expense of pastries for the firm is Rs 4.50 per dozen. At the end of the week the stale pastries are sold off for a lower price of Rs 3.50 per dozen. The overhead expenses attributable to pastry production are Rs 1.25 per dozen. Fresh pastries are sold in special boxes which expense 50 paise each and the stale pastries are sold wrapped in ordinary paper. The probability distribution of demand per week is as under:

Demand (in dozen)	: 0	1	2	3	4	5
Probability	0.01	0.14	0.2	0.5	0.1	0.05

Find the optimal production level of pastries per week.

9. Mohan Bagan, a local football club wants your advice on the number of programmes that should be printed for each game. The expense of printing and production of programmes for each game, as quoted by the local printer, is Rs 1,000 plus 4 paise per copy. Advertising revenue which has been agreed for the season represents Rs 800 for each game. Programmes are sold for 15 paise each. A review of sales during the previous seasons indicates that the following pattern is expected to be repeated during the coming season of 50 games:

Number of Programmes Sold	Number of Games
10,000	5
20,000	20
30,000	15
40,000	10

Programmes not sold at the game are sold as waste paper to a paper manufacturer at one paisa per copy. Assuming that the four options listed are the only possibilities (i) prepare

a payoff table, (ii) determine the number of programmes that would provide the largest profit, if a constant number of programmes were to be printed for each game,

(ii) calculate the profit which would arise from a perfect forecast of the number of programmes which would be sold at each game.

10. The probability distribution of monthly sales of an item of Ravi Ltd., is as follows:

Monthly sales (units) :	0	1	2	3	4	5	6
Probabilities :	0.01	0.06	0.25	0.30	0.22	0.10	0.06

The expense of carrying inventory (unsold during the month) is Rs 30 per unit per month and expense of unit shortage is Rs 70. Determine optimum stock to minimize expected expense.

11. Rajde, a modern home appliances dealer finds that the expense of holding a mini cooking range in stock for a month is Rs. 200 (insurance, minor deterioration, interest on borrowed capital, etc.). Customers who cannot obtain a cooking range immediately tend to go to other dealers and he esimates that for every customer who cannot get immediate delivery, he loses an average of Rs 500. The probabilities of a demand of 0, 1, 2, 3, 4, 5 mini cooking ranges in a month are 0.05, 0.10, 0.20, 0.30, 0.20, and 0.15 respectively. Determine the optimal stock level of cooking ranges. Also find EVPI.

12. Rajiv, a TV-dealer finds that the expense of holding a TV in stock for a week is Rs 50. Customers who cannot obtain .new TVs immediately tend to go to other dealers and he estimates that for every customer who cannot get immediate delivery he loses an average of Rs 200. For one particular model of TV the probabilities of a demand of 0,' 1, 2, 3, 4 and 5 TVs in a week are 0.05, 0.10, 0.20, 0.30, 0.20 and 0.15, respectively.

(i) How many televisions per week should the dealer order? Assume there is no time lag between ordering and delivery.

(ii) Compute EVPI.

(iii) The dealer is thinking of spending on a small market survey to obtain additional information regarding the demand levels. How much should he be willing to spend on such a survey?

13. Raman Industries Ltd. has a new product which they expect has great potential. At the moment they have two courses of action open to them: To test market (S1) and to drop product (S2).

If they test it, it will expense Rs 50,000 and the response could be positive or negative with probabilities 0.70 and 0.30, respectively. If it is positive, they could either market it with full effort or drop the product. If they market with full scale, then the result might be low, medium, or high demand and the respective net payoffs would be -Rs 1,00,000, Rs 1,00,000 or Rs 5,00,000. These outcomes have probabilities of 0.25, 0.55 and 0.20, respectively.

If the result of the test marketing is negative they have decided to drop the product. If, at any point, they drop the product there is a net gain of Rs 25,000 from the sale of scrap. All financial values have been discounted to the present. Draw a decision tree for the problem and indicate the most preferred decision.

14. Ajit, a manufacturing company has just developed a new product. On the basis of past experience, a product such as this will either be successful, with an expected gross

return of Rs 1,00,000, or unsuccessful, with an expected gross return of Rs 20,000. Similar products manufactured by the company have a record of being successful about 50 per cent of the time. The production and marketing expenses of the new product are expected to be Rs 50,000.

The company is considering whether to market this new product or to drop it. Before making its decision, a test marketing effort can be conducted at a expense of Rs 10,000. Based on past experience, test marketing results have been favourable about 70 per cent of the time. Furthermore, products favourably tested have been successful 80 percent of the time. However, when the test marketing result has been unfavourable, the product has only ' 'been successful 30 per cent of the time. What course of action should the company pursue?

15. The PQR Ltd., manufacture guaranteed tennis balls. At present time, approximately 10 per cent of the tennis balls are defective. A defective ball leaving the factory expenses the company Re 0.50 to honour its guarantee. Assume that all defective balls are returned. At a expense of Re 0.10 per ball, the company can conduct a test, which always correctly identifies both good and bad tennis balls.

(a) Draw a decision tree and determine the optimal course of action and its expected expense, (b) At what test expense the company should be indifferent to testing?

Unit :7 : Risk Analysis in Capital Budgeting

Introduction

The whole Activity of capital budgeting is related with the future activities like future cash outflow or future cash inflows etc. In fact various predictions are made regarding future inflows and outflows. Future is always uncertain so a project is selected on the basis of such uncertain future outcomes may cause problems if the actual picture differs from what was expected. It is possible to predict the outcome of some decisions with complete certainty because only one outcome can arise. However, there are many occasions when a decision can lead to more than one possible outcome, such situations are to be set with uncertainty. The traditional definition of the difference between risk and uncertainty has been that uncertainty cannot be quantified while risk can be, in this sense, risk is concerned with the use of quantification of the likelihood of future outcomes. The word uncertainly to cover all future outcomes which cannot be predicted with accuracy. People have different attitudes towards the future. Some welcome the opportunity to take risk and may be called risk takers or risk seekers. Other are risk averse.

An organization's performance is profoundly influenced by the elements contained with in its environment. There are so many internal and external factors which create uncertainty regarding future course of actions like political situation, inflation, state of economy etc. In turn the organization also has an impact on its environment. It is the role of a management to predict events that are likely to occur within the environment in order that the enterprise may meet any challenges or take advantage of any new opportunities.

Structure of the Chapter

7.1 Objectives:

By the end of this chapter the student will learn about

- How to make capital investment decision in uncertain conditions
- Different techniques to deal with uncertainty

7.2 Degrees of Certainty

The degrees of certainty may be classified into the following categories:

Complete certainty: All relevant information about the decision variables and outcomes is known with certainly.

Assumed certainty: For all practical purpose the future is known exactly and estimates become deterministic.

Risk: When it not known exactly what will happen in future, but the variance possibilities are neglected by their assumed probability of occurrence.

Uncertainty: Where a variety of outcomes are possible but probabilities cannot be assigned.

Extreme Uncertainty: Where no information is available to assess the likelihood of alternative outcomes.

Uncertainty arises from a lack of previous experience and knowledge. In a new venture, for example, it is possible for uncertainty to be attached to the following factors:

- Taxation rules.
- Level of operating expenses.
- Level sales volume.
- Level of revenue
- Level of selling prices.
- Level of capital outlay required.
- Date of completion.

Inevitably decision making under conditions of uncertainty is more complicated than is the case under risk condition. In fact there is no single best criterion that should be used in selecting a strategy of the various available techniques. Risk occurs where future outcomes of current action are unknown, but the probabilities of these future outcomes can be reasonably estimated from the knowledge of past and current events. Risk is therefore normally measured by volatility of returns because a certain outcome has no variance and hence, no volatility. Uncertainty on the other hand, occurs where the probabilities of future outcomes cannot be predicted from past or current events, because no probability estimates are available.

7.3 Techniques to Deal with Risk

Risk Adjusted Discount Rate:

The easiest and most common method of allowing for risk is by adjusting the discount rate applied to the future cash flows arising from the project. The rate of discount is equal to the expense of capital or in other words it is the rate, which the investors demand from the company on their investments. By this method a premium can

be added to the average required discount rate as a safety margin to compensate for the enhanced risk of the project. This acknowledges that if the same discount rate is applied to all proposed capital projects, no distinction would be made between high and low risk projects. It means that higher the risk in a project, higher would be the discount rate and the lower the risk in the project, the lower would be the discount rate.

Suppose, the rate of interest on government treasury bills is 8% p.a. then the risk free interest rate is 8%. If the company is considering a project of producing a new product to put in the existing market, then due to risk, they would require a premium of say, 6%, then the risk adjusted discount rate would be 8%+6% = 14%.

However, the difficulty in this method is that it is difficult to determine the risk adjusted discount rate. How high should be the discount rate or what should be the premium over risk free rate in respect of a project involving higher risk and what is the degree of risk involved in the project.

Certainty Equivalent Coefficient:

Certainty equivalent method overcomes the defects contained in the method risk adjusted discount rate. Here in this method future cash flows are estimated as usual, but along with that risk less cash flow is also determined. Suppose the management estimates that taking into account the risk involved in the project, the cash flow for third year is Rs.80000. But they believe that a minimum of Rs.64000 is sure to be received without risk. Thus the risk less flow of third year is Rs.64000. If we find the ratio, it will give certainty equivalent coefficient.

$$\text{Certainty Equivalent coefficient} = \frac{\text{Risk less cash flow}}{\text{Risky cash flow}}$$

$$= \frac{64000}{80000}$$

$$= 0.8$$

Thus in the above manner coefficients are worked out for all the years of life of the project. When risky cash flows are multiplied by these coefficients, we get risk less returns. The rate of discount used for this purpose is the risk free rate and not adjusted rate. If the net present value of this risk less cash flows is positive then the project is accepted.

Pay Back Criteria:

However, another way of allowing for risk can be using the payback evaluation technique in either its simple form or preferably by calculating the discounted payback for the projects. The company can set a maximum period for the project to repay its original investment and this also reflects the risk profile of the project.

Sensitivity Analysis

Sensitivity analysis is the study of the key assumptions or calculations on which a management decision is based in order to predict alternative outcomes of that decision if different assumptions are adopted. It is a 'what if' technique that measure how the expected values in a decision model will be affected by changes in the data. Sensitivity analysis or 'what if' involve evaluating the impact on the financial returns from the project if certain key variables changed from those forecast in the base evaluation. The sensitivity of the overall returns from the project to relatively small changes in one or a few, key variables helps managers to understand the risk profile of the project, if these variables are also non-controllable the level of risk may be unacceptably high and the project is rejected.

Sensitivity analysis is a modeling procedure used in forecasting whereby changes are made in the estimates of the variables to establish whether any will critically affect the outcome of the forecast. It is possible to use sensitivity analysis for helping to determine the value of information in addition to its role in strategic decision making. Sensitivity analysis seeks to determine the range of variations in the coefficients over which the solution will remain optimal.

Probability Assignment:

In every method it is seen that it gives different estimates of cash flows but fails to show the chance of variability of these cash flow. For this purpose probability may be assigned to each of the cash flows. The probability assignment will give some definite measure of possibility of different cash flows. It indicates the percentage chance of occurrence of each possible cash flow.

Simulation

Simulation is the representation of a system by a model which will react to change in a similar way to that which is being simulated. This evolves a decision maker to predict the outcome of particular decision through testing it via the model. Normally simulation techniques are used to solve problems involving uncertainty. There are several techniques of simulation that are in use. However, 'Monte-Carlo' method is very popular as it is very simple and easy to use. The technique uses random numbers and is used to solve problems which involves conditions of uncertainty.

In simulation, a computer would normally be used to build and run the model. This is particularly important in this area, since meaningful information can be extracted from the simulation only after a number of runs with different random number.

Some problems are too complex to solve with pure mathematics, or they involve random elements or risk situation that defy a practical mathematical solution. In such situation, analysts sometime construct a model of the real world problem and use a trail and error approach to arrive at reasonable solutions to the problem. For using simulation one should go through the following steps:

- Decide what course of action to take.
- Construct a numerical model.

- Run the experiment.
- Consider the results and the possibilities to modify the model or changes data inputs.
- Set up possible course of action for testing.
- Introduce the variables associated with the problems.
- Define the problem precisely.

Simulation modeling is extremely useful in production Scheduling manpower planning decision, parking problems, inventory problems, investment analysis, Queueing problems maintenance problems, testing a series of marketing problems, location of factories for expense reduction etc.

Optimistic – Pessimistic Estimates

In decision-making the first step is usually to make a single 'best estimate' for each item. One might then also make optimistic and pessimistic estimates for each variable. Another approach is to make the 'most likely' estimate for each item in turn, to see how much difference it makes to the overall result. Large changes to particular items will often not be important, so we need to identify those critical variables where even a fairly small change can make quite a large difference to the overall result.

The worst possible/best possible outcomes can be evaluated from the pessimistic and optimistic attitudes of the decisions made. In making decision under uncertainty, the decision maker should assess not only the most likely outcome from a decision but also the outcome that will arise if the worst possible happens. This analysis will help in understanding the full range of possible outcomes from a decision and will help the decision maker to take right decision keeping in view the risk involved in the decision.

Suppose, the investment in a project is Rs.75000 and its useful life is 5 years. The expense of the project is 12%. The optimistic future cash flow is estimated at Rs.34000 and the present value of annuity of Rs.1 for 5 years at 12% is Rs.3.605, in pessimistic estimation, the cash flow is Rs.17000 per annum, while it is Rs.23000 under normal or most likely conditions. The net present value under all the three estimates would be as follows.

Optimistic = 34000 * 3.605 = 122570
Most likely = 23000 * 3.605 = 82915
Pessimistic = 17000 * 3.605 = 61285

Thus this method takes into account the uncertainty of future and gives three estimates. It is thus superior to the single figure forecasts.

Standard Deviation in Measurement of Risk

Risk is measured by the possible variation of outcomes around the expected value and the decision will be taken keeping in view the variation in the expected value where two projects have the same expected value the decision maker would choose the project which has smaller variation in expected value. So, the project which has a larger standard deviation will be considered as more risky as compared to a project having smaller standard deviation. Standard deviation is the square root of variance.

Coefficient of Variation

The standard deviation is an absolute measure of variability and not the relative measure of variability and so it can not be useful in comparing the projects when the size of such projects are different. In such circumstances, a useful measure of risk for project comparison is the coefficient of variation which is calculated as follows:

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{EV of profit}} \times 100$$

EV = Expected Value

A project with a higher coefficient of variation would be more risky than a project with a lower coefficient of variation.

Value of Information

We can sometimes reduce the uncertainty involved in making a decision by collecting more information. However, we will usually have to pay for this additional information, the value of perfect information tells us the maximum amounts it is worth paying for it. If we know in advance which one of the outcome will occur, then we choose the decision which all lead to the maximum payoff. This does not mean that we can control the choice of outcome. Outcome will be the same whether favorable or unfavorable but we can make an estimate of it.

Let us take the illustration. The bakery shopkeeper could take orders for breads to be delivered the following day. He cannot control how many orders he will receive but he could earn on an average Rs.5 per day without having any kind of information regarding the demand of the breads. Sometimes due to lack of information it may happen that he has more breads than the demand and sometimes situation may be reverse but then he found a source from which he could make out perfact estimation regarding demand of his bread but for providing such information to him he had to pay certain amount to the information provider. Buy getting such perfact information he was able to generate maximum profit of Rs.8 per day and this profit is certain to be received.

The difference between this figure and the maximum expected pay off without perfect information is called "The value of perfect Information".

The value of perfect information for the shopkeeper is: Rs.8 – Rs.5 = Re.3 per day, which is equal to the minimum expected opportunity loss.

The value of perfect information means the maximum amount we should pay for additional information about the likelihood of each outcome arising. The shopkeeper could afford to pay up to Rs.3 per day for getting perfact information or to operate an ordering systems or for market research information to enable the daily demand to be predicted more accurately than at present.

The information received may be of two types:

Perfect Information

Perfect information is information about the future outcome of an event with absolute certainty and is guaranteed to predict the future with 100% accuracy. Perfect information, therefore, removes all doubts and uncertainty from decision, and it would enable managers to make decisions with complete confidence that they have selected the most profitable course of action.

Imperfect Information:

There is one serious drawback to the technique we must take care off. Estimating the value of perfect information should help management to decide whether obtaining information would be worth the expense of its collection, but in practice information is rarely ever perfect. Market still be wrong; they provide imperfect information. It is possible, however, to arrive at an assessment of how much it would be worth paying for such imperfect information, given that we have a rough indication of how right or wrong it is likely to be.

Information, whether perfect or imperfect, will expense money to obtain and so if the option exists for a decision maker to obtain the information or not, a further decision that has to be made is: 'would the information be worth the expense of obtaining it?'

7.4 Practicals :

1. The total investment in a project of Vinee Ltd. is estimated at Rs. 110000. Its forecast cash flow and certainty equivalent are as follows:

Year	Cash Flow (Rs.)	Co-efficient
1	55000	0.90
2	40000	0.70
3	40000	0.50
4	60000	0.30

The risk free discount rate 10%. Determine on the basis of NPV, whether the project should be accepted. The present values of Rs.1 at 10% for different year are 0.909, 0.826, 0.751, 0.683.

Solution:

Year	Cash flow	Co-efficient	Risk less Cash flow	Discount factor	Discounted Cash flow
1	55000	0.90	49500	0.909	44995
2	40000	0.70	28000	0.826	23130
3	40000	0.50	20000	0.751	15120
4	60000	0.30	18000	0.683	12290
					95535
				Investment	110000
				Net Present Value	–14464

As the NPV of the project is negative, is should no be accepted.

2. A Vipul Ltd. is considering a proposal to buy one machine out of the two. An investment of Rs. 55000 is required in each machine and useful life of each machine is estimated at 4 year. The vendors of these machines have given a guarantee to purchase these machines for Rs. 10000 at the and of their useful life. The company uses certainty-equivalent co-efficient to evaluate the risky project. The risk-adjusted rate of discount is 16%, while the risk less discount rate is 10%.

	Machine A		Machine B	
Year	**Cash flow**	**C.E.**	**Cash flow**	**C.E.**
1	30000	0.8	18000	0.9
2	30000	0.7	36000	0.8
3	30000	0.6	24000	0.7
4	30000	0.5	32000	0.4

Which machine should be purchased?

Solution:

When certainty equivalent coefficient is used, it means that the cash flow arrived at on that basis certain to be received e.g. the cash flow for first year for machine A is estimated at Rs. 30000 and its C.E. is 0.8. It means that Rs. 30000 x 0.8 = Rs. 24000 cash flow is sure to be received during the first year. There is no risk about it. Its present value should be calculated at risk less discount rate, because risk has been covered by discounting the cash flows. So discount rate should not be adjusted for risk.

Remember that when C.E. is used for adjusting cash flow forecast, always use risk less discount rate.

Machine A:

Year	**Cash Flow**	**Certainty Equivalent**	**Revised Cash flow**	**Discount Factor**	**Present Value**
1	30000	0.8	24000	0.909	21716
2	30000	0.7	21000	0.826	17346
3	30000	0.6	18000	0.751	13518
4	30000	0.5	15000	0.683	10245
Value at the end	10000	1.0	10000	0.683	6830
					69655
			Total Present Value		
				Less: Investment	55000
				Net Present Value	+ 14655

Machine B:

Year	Cash Flow	Certainty Equivalent	Revised Cash flow	Discount Factor	Present Value
1	18000	0.9	16200	0.909	14726
2	36000	0.8	28800	0.826	23789
3	24000	0.7	16800	0.71	12617
4	32000	0.4	12800	0.683	8742
Value at the end	10000	1.0	10000	0.683	6830
			Total Present Value		66704
			Less: Investment		55000
			Net Present Value		+ 11704

As in both the cases the value to be realized is Rs. 10000 at the end, it is certain to be received and so its C.E. should be treated as 1.

Though both the alternative presents positive NPVs but project A is preferable because it produces higher NPV than project B.

3. A Nishu Ltd. is considering two mutually exclusive projects A and B. In both the cases, initial investment will be Rs. 1,10,000 and the useful life of both will be 10 years. No projects has no scrap value. The probable cash flow will be as follows:

	Project A Rs.	Project B Rs.
Optimistic	50,000	70,000
Most Likely	40,000	35,000
Pessimistic	18,000	4,000

If the rate of discount is 10% calculate the present value and state which project is better out of the two. The annuity of Re. 1 at 10% for 10 years is Rs. 6.145.

Solution:

	Project A	Net Present Value	Project B	Net Present Value
Optimistic	50,000x6.145 =307250	1,97250	70,000 x 6.145 = 430150	320150
Most likely	40,000x6.145 =245800	135800	35000 x 6.145 = 215075	105075
Pessimistic	18,000 x 6.145 =Rs. 1,10,610	610	4000 x 6.145 = 24580	-85420

Comparing the two projects, it seems project B is more risky, as in the

pessimistic estimate it may result into a loss of Rs. 85,420, while in case if it materializes, it may result into a considerable profit. Thus project A should be selected. However it depends upon the attitude of the decision maker. If he is conservative, he will selected project A as there is no possibility of loss. But if he is a risk taker, he will select B as it has a possibility of paying a very high return in case of success.

4. A Parshwa Ltd. is considering a project in which investment in fixed assets would be Rs. 48 lakhs and investment in current assets would be Rs. 22 lakhs. Its useful life is estimated at 5 years, during which returns would be earned at equal rate and at the end of which the scrap value will be Rs. 3 lakhs. The estimate of it income before depreciation and taxes will be as follow:

Annual Income (In lacs Rs.)	Probability
5	0.1
10	0.2
20	0.5
30	0.1
40	0.1

Tax rate is 50 %. The minimum rate of return is 12 %. From the tables, it is found that the present value of Re. 1 for 10 years at 12 % rate of discount would be Rs. 5.670 and its present value at the end of $10t^h$ year would be 0.322. Would, you recommend this proposal?

Solution:

The income given in the example is before depreciation and taxes hence we shall first calculate depreciation.

Expense of fixed assets Rs. 48 lacs – scrap value of Rs. 3 lacs
= Rs. 45 lacs = Depreciation on Straight line method = Rs. 45 lacs / 10 years = Rs. 4.5 lacs.

Cash Flow (Before tax and Depreciation	Depre ciation	Taxable Income	Tax at 50 %	Income After tax	Cash Flow Including Depreciat ion	Probabi lity	Adjusted Revised Cash flow
Rs. 5 lacs	4.5	0.5	0.25	0.25	4.75	0.1	0.475
Rs. 10 lacs	4.5	5.5	2.75	2.75	7.25	0.2	1.450
Rs. 20 lacs	4.5	15.5	7.75	7.75	12.25	0.5	6.125
Rs. 30 lacs	4.5	25.5	12.75	12.75	17.25	1.0	1.725
Rs. 40 lacs	4.5	35.5	17.75	17.75	22.25	0.1	2,225
							12.00

Calculation for Net Present Value	Rs.
Rs. 12 lcas for 10 years x 5.670 (at 12%) =	68.04 lacs
Rs. 3 lacs scrap value at the end of 10^{th} years x 0.322 =	.966

Current Assets: at the end of 10^{th} years Rs. 22 lacs x 0.322		7.084
Total present value of cash flow		76.09
- total investment (Rs48+ Rs. 22 lacs)		70.00
	Net Present Value	+6.09

This proposal may be accepted because its NPV is positive.

Note: working capital blocked in the business will be treated as outflow initially and at the end of the project it will be released from the project and in that year it will be treated as inflow for the project. In the same manner, scrap value has two uses, initially it is used for calculation of amount of depreciation and in the last year of the project it is treated as inflow in the project.

7.5 Exercise:

Answer the following questions

1. What are the reasons for which risk reducing measures to be used in capital budgeting?
2. Explain sensitivity analysis as a risk reducing measure in capital budgeting?
3. Explain simulation as a risk reducing measure in capital budgeting?
4. What a maximum amount can be paid for obtaining perfect information for forth coming activities?

7.6 Practical exercise

1. The total investment in a project of Vinee Ltd. is estimated at Rs. 110000. Its forecast cash flow and certainty equivalent are as follows:

Year	Cash Flow (Rs.)	Co-efficient
1	55000	0.90
2	40000	0.70
3	40000	0.50
4	60000	0.30

The risk free discount rate 10%. Determine on the basis of NPV, whether the project should be accepted. The present values of Rs.1 at 10% for different year are 0.909, 0.826, 0.751, 0.683.

Solution:

Year	Cash flow	Co-efficient	Risk less Cash flow	Discount factor	Discounted Cash flow
1	55000	0.90	49500	0.909	44995
2	40000	0.70	28000	0.826	23130
3	40000	0.50	20000	0.751	15120
4	60000	0.30	18000	0.683	12290
					95535
				Investment	110000
				Net Present Value	–14464

As the NPV of the project is negative, is should no be accepted.

2. A Vipul Ltd. is considering a proposal to buy one machine out of the two. An investment of Rs. 55000 is required in each machine and useful life of each machine is estimated at 4 year. The vendors of these machines have given a guarantee to purchase these machines for Rs. 10000 at the and of their useful life. The company uses certainty-equivalent co-efficient to evaluate the risky project. The risk-adjusted rate of discount is 16%, while the risk less discount rate is 10%.

	Machine A		Machine B	
Year	**Cash flow**	**C.E.**	**Cash flow**	**C.E.**
1	30000	0.8	18000	0.9
2	30000	0.7	36000	0.8
3	30000	0.6	24000	0.7
4	30000	0.5	32000	0.4

Which machine should be purchased?

Solution:

When certainty equivalent coefficient is used, it means that the cash flow arrived at on that basis certain to be received e.g. the cash flow for first year for machine A is estimated at Rs. 30000 and its C.E. is 0.8. It means that Rs. 30000 x 0.8 = Rs. 24000 cash flow is sure to be received during the first year. There is no risk about it. Its present value should be calculated at risk less discount rate, because risk has been covered by discounting the cash flows. So discount rate should not be adjusted for risk.

Remember that when C.E. is used for adjusting cash flow forecast, always use risk less discount rate.

Machine A:					
Year	**Cash Flow**	**Certainty Equivalent**	**Revised Cash flow**	**Discount Factor**	**Present Value**
1	30000	0.8	24000	0.909	21716
2	30000	0.7	21000	0.826	17346
3	30000	0.6	18000	0.751	13518
4	30000	0.5	15000	0.683	10245
Value at the end	10000	1.0	10000	0.683	6830
			Total Present Value		69655
				Less: Investment	55000
				Net Present Value	+ 14655

Machine B:					
Year	**Cash** Flow	Certainty Equivalent	Revised Cash flow	**Discount** Factor	Present Value
1	18000	0.9	16200	0.909	14726
2	36000	0.8	28800	0.826	23789
3	24000	0.7	16800	0.71	12617
4	32000	0.4	12800	0.683	8742
Value at the end	10000	1.0	10000	0.683	6830
			Total Present Value		66704
			Less: Investment		55000
			Net Present Value		+ 11704

As in both the cases the value to be realized is Rs. 10000 at the end, it is certain to be received and so its C.E. should be treated as 1.

Though both the alternative presents positive NPVs but project A is preferable because it produces higher NPV than project B.

3. A Nishu Ltd. is considering two mutually exclusive projects A and B. In both the cases, initial investment will be Rs. 1,10,000 and the useful life of both will be 10 years. No projects has no scrap value. The probable cash flow will be as follows:

	Project A Rs.	Project B Rs.
Optimistic	50,000	70,000
Most Likely	40,000	35,000
Pessimistic	18,000	4,000

If the rate of discount is 10% calculate the present value and state which project is better out of the two. The annuity of Re. 1 at 10% for 10 years is Rs. 6.145.

Solution:

	Project A	**Net Present Value**	**Project B**	**Net Present Value**
Optimistic	50,000x6.145		70,000 x 6.145	
	=307250	1,97250	= 430150	320150
Most likely	40,000x6.145		35000 x 6.145	
	=245800	135800	= 215075	105075
Pessimistic	18,000 x 6.145		4000 x 6.145	
	=Rs. 1,10,610	610	= 24580	-85420

Comparing the two projects, it seems project B is more risky, as in the pessimistic estimate it may result into a loss of Rs. 85,420, while in case if it materializes, it may result into a considerable profit. Thus project A should be selected. However it depends upon the attitude of the decision maker. If he is conservative, he will selected project A as there is no possibility of loss. But if he is a risk taker, he will select B as it has a possibility of paying a very high return in case of success.

4. A Parshwa Ltd. is considering a project in which investment in fixed assets would be Rs. 48 lakhs and investment in current assets would be Rs. 22 lakhs. Its useful life is estimated at 5 years, during which returns would be earned at equal rate and at the end of which the scrap value will be Rs. 3 lakhs. The estimate of it income before depreciation and taxes will be as follow:

Annual Income (In lacs Rs.)	Probability
6	0.1
11	0.2
21	0.5
31	0.1
40	0.1

Tax rate is 50 %. The minimum rate of return is 12 %. From the tables, it is

found that the present value of Re. 1 for 10 years at 12 % rate of discount would be Rs. 5.670 and its present value at the end of $10t^h$ year would be 0.322. Would, you recommend this proposal?

Solution:

The income given in the example is before depreciation and taxes hence we shall first calculate depreciation.

Cost of fixed assets Rs. 48 lacs – scrap value of Rs. 3 lacs
= Rs. 45 lacs = Depreciation on Straight line method = Rs. 45 lacs / 10 years = Rs. 4.5 lacs.

Cash Flow (Before tax and Depreciation	Depre ciation	Taxable Income	Tax at 50 %	Income After tax	Cash Flow Including Depreciat ion	Probabi lity	Adjusted Revised Cash flow
Rs. 5 lacs	4.5	0.5	0.25	0.25	4.75	0.1	0.475
Rs. 10 lacs	4.5	5.5	2.75	2.75	7.25	0.2	1.450
Rs. 20 lacs	4.5	15.5	7.75	7.75	12.25	0.5	6.125
Rs. 30 lacs	4.5	25.5	12.75	12.75	17.25	1.0	1.725
Rs. 40 lacs	4.5	35.5	17.75	17.75	22.25	0.1	2,225
							12.00

Calculation for Net Present Value	Rs.
Rs. 12 lcas for 10 years x 5.670 (at 12%) =	68.04 lacs
Rs. 3 lacs scrap value at the end of 10^{th} years x 0.322 =	.966
Current Assets: at the end of 10^{th} years Rs. 22 lacs x 0.322	7.084
Total present value of cash flow	76.09
- total investment (Rs48+ Rs. 22 lacs)	70.00
Net Present Value +6.09	

This proposal may be accepted because its NPV is positive.

Note: working capital blocked in the business will be treated as outflow initially and at the end of the project it will be released from the project and in that year it will be treated as inflow for the project. In the same manner, scrap value has two uses, initially it is used for calculation of amount of depreciation and in the last year of the project it is treated as inflow in the project.

7.7 Exercise

1. Mahavir Ltd. is considering to purchase one out of the two machines for production of a new product. The investment in each machine will be Rs. 1,10,000 and they would give benefit of 12 years. For each alternative, three estimates of cash flows are given: Most likely, Optimistic and Pessimistic.

	Machine A Rs.	Machine B Rs.
Cost Price	1,10,000	1,10,000
Estimate of Cash flow:		
Optimistic	33,000	44,000
Most Likely	25,000	20,000
Pessimistic	19,000	1,000

The cost of capital of the project is 14%. Which project is more risky? The annuity of Rs. 1 at 14% for 12 years is Rs. 5.66.

2. Two mutually exclusive projects are under consideration by Harry Ltd. The initial investment in both of them is Rs. 1,00,000. The economic life of both is estimated at 5 years and they have no scarp value. Their estimated cash flows and certainty equivalent are as under:

Year	Project A Cash flow Rs.	Project B Certainty Equivalent	Cash flow	Certainty Equivalent
1	70,000	0.8	85,000	0.5
2	65,000	0.6	1,25,000	0.4
3	55,000	0.5	50,000	0.3
4	35,000	0.4	30,000	0.2
5	30,000	0.3	20,000	0.1

If the cost of capital of both is 15 %, calculate the net present value of both and state which projects is acceptable.

3. The Cautious Ltd is considering a proposal for the purchase of a new machine requiring an outlay of Rs 1,500 lakh. Its estimate of the cash flow distribution for the three-year life of the machine is given below (amount in Rs lakh):

Period -1		Period 2		Period 3	
Cash flows	Probability	Cash flows	Probability	Cash flows	Probability
Rs 800	0.1	Rs 800	0.1	Rs 1,200	0.2
650 .	0.2	750	0.3	900	0.5
400	0.4	600	0.4	600	0.2
200	0.3	500	0.2	300	0.1

The probability distribution is assumed to be independent. Risk-free rate of interest is 5 per cent. From the above information, determine the following:

(i) the expected NPV of the project;

(ii) the standard deviation of the probability distribution of NPV; (iii) the probability that the NPV will be

(a) zero or less (assuming that the distribution is normal);

(b) greater than zero; and

(c) at least equal to the mean;

(iv) the profitability index of the expected value; and

(v) the probability that the profitability index will be less than 1.

4. A company employs certainty-equivalent approach in the evaluation of risky investments. The capital budgeting department of the company has developed the following information regarding a new project:

Year	Expected CFAT (Rs thoudsand)	Certainty-equivalent quotient
0	250	1.0
1	160	0.8
2	140	0.7
3	120	0.6
4	120	0.4
5	80	0.3

The firm's cost of equity capital is 18 per cent; its cost of debt is 9 per cent and the riskless rate of interest in the market on the government securities is 6 per cent. Should the project be accepted?

5. The Delta Corporation is considering an investment in one of the two mutually exclusive proposals: Project A which involves an initial outlay of Rs 1,70,000 and Project B which has an outlay of Rs. 1,56,000. The Certainty-

Equivalent Approach is employed in evaluating risky investments. The current yield on treasury bills is 0.05 and the company uses this as the riskless rate. The expected values of net cash flows with their respective certainty-equivalents are:

Year	Project A		Project B	
	Cash flow (Rs thousand)	Certainty-equivalent	Cash flows (Rs thousand)	Certainty-equivalent
1	95	0.8	90	0.9
2	100	0.7	90	0.8
3	110	0.5	100	0.6

(i) Which project should be acceptable to the company?

(ii) Which project is riskier? How do you know?

(iii) If the company was to use the risk-adjusted discount rate method, which project would be analysed with .higher rate?

7. A company has under consideration two mutually exclusive projects for increasing its plant capacity. The management has developed pessimistic, most likely and optimistic estimates of the annual cash flows associated with each project. The estimates are as follows:

	Project A	Project B
Net investment	Rs 32,000	Rs 30,000
CFAT estimates:		
Pessimistic	1,200	3,700
Most 'likely	4,000	4.000
Optimistic	7,000	4,500

(a) Determine the NPV associated with each estimate given for both the projects. The projects have 20 year life each and the firm's cost of capital, 10 per cent.

(b) Which project do you consider should be selected by the company and why?

8. A company is examining two mutually exclusive investment proposals. The management of the company uses 'certainty-equivalents (CE) to evaluate new investment proposals. From the following information pertaining to these projects advise the company which project should be taken up by the company:

Year	Proposal A		Proposal B	
	CFAT	CE	CFAT	CE
0	Rs 25,500	1.0	Rs 25,000	1.0
1	15,000	0.8	9,000	0.9
2	15,000	0.7	18,000	0.8
3	15,000	0.6	12,000	0.7
4	15,000	0.5	16,000	0.4

The firm's cost of capital is 12 per cent and risk-free borrowing rate is 6 per cent.

9. A company is. considering a proposal to buy one of the two machines to manufacture a new commodity. Each of the machines requires investments of Rs 50,000 and is expected to provide benefits over a period of 12 years. The firm has made 'pessimistic,' 'most likely' and 'optimistic' estimates of the returns associated with each of these alternatives. These estimates are as follows:

	Machine A	Machine B
Cost	Rs 50,000	Rs 50,000
Cash flow estimates:		
Pessimistic	8,500	0
Most likely	12,500	10,000
Optimistic	16,000	20,000

Assuming 14 per cent cost of capital, which project do you consider more risky, and why?

10. Company is considering two mutually exclusive projects X and Y. Project X costs Rs 32,000 and Project Y Rs 36,000. You have been given below the net present value probability distribution for each project:

Project X		Project Y	
NPV estimate	Probability	NPV estimate	Probability
Rs 4,000	0.1	Rs 3,000	0.2
6,000	0.4	6,000	0.3
12,000	0.4	12,000	0.3
15,000	0.1	15,000	0.2

(a) Compute the expected net present value of projects X and Y.

(b) Compute the risk attached to each project that is, standard deviation of each probability distribution.

(c) Which project do you consider more risky and why?

(d) Compute the profitability index of each project.

11. The probability distributions of two projects' NPV are given below:

Project X		Project Y	
NPV	Probability	NPV	Probability
Rs 6,000	0.2	0	0.1
7,500	0.7	Rs 7,500	0.7
10,000	0.1	15,000	0.2

Calculate the expected value, the standard deviation, and the coefficient of variation for each project. Which of these mutually exclusive projects do you prefer and why?

12. A company is considering a proposal to purchase a new machine. The machine has an initial cost of Rs 55,000. The capital budgeting department has developed the following discrete probability distribution for cash flows generated by the project during its useful life of 3 years:

Period 1		Period 2		Period 3	
CFAT	Probability	CFAT	Probability	CFAT	Probability
Rs 16,000	0.2	Rs 20,000	0.5	Rs 25,000	0.1
20,000	0.4	23,000	0.1	30,000	0.3
25,000	0.3	25,000	0.2	35,000	0.3
30,000	0.1	28,000	0.2	50,000	0.3

(a) Assuming that the probability distributions of cash flows for future periods are independent, the firm's cost of capital is 10 per cent and the firm can invest in 5 per cent treasury bills, determine the expected NPV.

(b) Determine the standard deviation about the expected value.

(c) If the total distribution is approximately normal and assumed continuous,

(i) what is the probability of the NPV being less than zero,

(ii) greater than zero,

(iii) profitability index being 1 or less,

(iv) at least equal to mean,

(v) 10 per cent below mean, and

(vi) 10 per cent above mean?

13. Determine the risk-adjusted net present value of the following projects:

Net cash outlays (Rs)	1,00,000	1,20,000	2,10,000
Project life (years)	5	5	5
Annual cash inflow (Rs)	32,000	42,000	70,000
Coefficient of variation	0.4	0.8	1.2

The company selects the risk-adjusted rate of discount on the basis of the coefficient of variation:

Coefficient of variation	Risk-adjusted rate of discount
0.0	0.10
0.4	0.12
0.8	0.14
1.2	0.16
1.6	0.18
2.0	0.22
More than 2.0	0.25